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LETTERS TO THE EDITOR

COMMENTS ON "A NUMERICAL STUDY ON THE PROPAGATION OF SOUND THROUGH CAPILLARY TUBES WITH MEAN FLOW"

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Jeong and Ih [1] have investigated the sound propagation through a capillary duct with mean flow. Numerical solutions have been found for non-isentropic acoustic disturbances with inclusion of the radial velocity terms in the governing equations and the radial velocity profiles over a cross-section varying the shear wave number which have been calculated for both the case of zero mean flow and the case of the presence of steady flow in the capillary duct. However, for both of these cases the profiles mentioned above do not satisfy equation (7) of reference [1] at the wall of the duct ($\eta = 1$), as will be shown here.

Indeed, equation (7) of reference [1] that applies at $\eta = 1$ is given by

$$\frac{\mathrm{d}v}{\mathrm{d}\eta_{|\eta=1}} = \left[-k(\mathrm{i}\rho + \Gamma u + M_0 \Gamma \rho) - \frac{v}{\eta} \right]_{|\eta=1}, \tag{1}$$

where

$$M_0 = 2\bar{M}(1-\eta^2), \quad \bar{M} = \int_0^1 M(\eta) 2\eta \, \mathrm{d}\eta, \quad \Gamma = \Gamma_1 + \mathrm{i}\Gamma_2, \quad \eta = r/R, \tag{2}$$

u, v represent the perturbed axial and radial velocities, respectively, $k = \omega R/\bar{a}$ is the reduced frequency, ω is the frequency of harmonic acoustic disturbances, $\bar{a}$ is the speed of sound in the isentropic condition without fluid flow, R is the tube radius, Γ means the normalized complex propagation constant [2], ρ is the perturbed density, $\bar{M}$ is Mach number, $\bar{M} = u/\bar{a}$, r denotes radial co-ordinate, i is the imaginary unit.

In accordance with reference [1], boundary conditions at the wall ($\eta = 1$) are

$$u = 0, \quad v = 0, \quad M_0 \equiv 2\bar{M}(1-1) = 0, \quad T = 0, \tag{3}$$

where T is the perturbed temperature.

On substituting for T from equations (3) into equation (10) of reference [1] one obtains

$$\rho(\eta = 1) = p(\eta = 1) - T(\eta = 1) = p(\eta = 1), \tag{4}$$

where p is the perturbed pressure.

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After application of the boundary conditions (3) and (4), equation (1) is equivalent to

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{kp}\right)_{|\eta=1} = -\,\mathrm{i}. \tag{5}$$

Note that the real part of the function $\mathrm{d}/\mathrm{d}\eta\,(v/kp)$ is equal to zero at $\eta = 1$ and hence, the real part of the function (v/kp) will behave like

$$\mathrm{Re}\left(\frac{v}{kp}\right) \equiv f_1(\eta) \to a_2(\eta-1)^2 + a_3(\eta-1)^3 + \cdots \qquad \text{as } \eta \to 1, \tag{6}$$

while the imaginary part of the function (v/kp) will behave like

$$\mathrm{Im}\left(\frac{v}{kp}\right) \equiv f_2(\eta) \to -(\eta-1) + b_2(\eta-1)^2 + b_3(\eta-1)^3 + \cdots \qquad \text{as } \eta \to 1, \tag{7}$$

where a_n, b_n are constants, $n = 1, 2, 3, \ldots$.

Now one can show that

$$\left|\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{kp}\right)\right| = \left|\frac{\mathrm{d}}{\mathrm{d}\eta}\left|\left(\frac{v}{kp}\right)\right|\right| \qquad \text{at } \eta = 1.$$

Indeed, by definition,

$$\left|\left(\frac{v}{kp}\right)\right| = |f_1(\eta) + \mathrm{i}f_2(\eta)| = \sqrt{f_1^2(\eta) + f_2^2(\eta)}. \tag{8}$$

From this result and equations (6) and (7) it follows that

$$\left|\frac{\mathrm{d}}{\mathrm{d}\eta}\left|\left(\frac{v}{kp}\right)\right|\right|_{|\eta=1} = \underset{\eta\to 1}{\mathrm{Lt}}\left|\frac{\mathrm{d}}{\mathrm{d}\eta}\left|\left(\frac{v}{kp}\right)\right|\right| = \underset{\eta\to 1}{\mathrm{Lt}}\left|\frac{f_1(\eta)\,\mathrm{d}f_1(\eta)/\mathrm{d}\eta + f_2(\eta)\,\mathrm{d}f_2(\eta)/\mathrm{d}\eta}{\sqrt{f_1^2(\eta) + f_2^2(\eta)}}\right|$$

$$= \underset{\eta\to 1}{\mathrm{Lt}}\left|\frac{\dfrac{f_1(\eta)}{f_2(\eta)}\dfrac{\mathrm{d}f_1(\eta)}{d\eta} + \dfrac{\mathrm{d}f_2(\eta)}{\mathrm{d}\eta}}{\sqrt{1 + \dfrac{f_1^2(\eta)}{f_2^2(\eta)}}}\right|$$

$$= \underset{\eta\to 1}{\mathrm{Lt}}\left|\frac{\dfrac{a_2(\eta-1)^2 + a_3(\eta-1)^3 + \cdots}{-(\eta-1) + b_2(\eta-1)^2 + b_3(\eta-1)^3 + \cdots}\dfrac{\mathrm{d}f_1(\eta)}{\mathrm{d}\eta} + \dfrac{\mathrm{d}f_2(\eta)}{\mathrm{d}\eta}}{\sqrt{1 + \left[\dfrac{a_2(\eta-1)^2 + a_3(\eta-1)^3 + \cdots}{-(\eta-1) + b_2(\eta-1)^2 + b_3(\eta-1)^3 + \cdots}\right]^2}}\right|$$

$$= \underset{\eta\to 1}{\mathrm{Lt}}\left|\frac{\dfrac{a_2(\eta-1) + a_3(\eta-1)^2 + \cdots}{-1 + b_2(\eta-1) + b_3(\eta-1)^2 + \cdots}\dfrac{\mathrm{d}f_1(\eta)}{\mathrm{d}\eta} + \dfrac{\mathrm{d}f_2(\eta)}{\mathrm{d}\eta}}{\sqrt{1 + \left[\dfrac{a_2(\eta-1) + a_3(\eta-1)^2 + \cdots}{-1 + b_2(\eta-1) + b_3(\eta-1)^2 + \cdots}\right]^2}}\right| = \left|\frac{\mathrm{d}f_2(\eta)}{\mathrm{d}\eta}\right|_{|\eta=1}. \tag{9}$$

Next, by definition,

$$\left|\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{kp}\right)\right|_{|\eta=1} = \left|\frac{\mathrm{d}}{\mathrm{d}\eta}[f_1(\eta)+\mathrm{i}f_2(\eta)]\right|_{|\eta=1} = \sqrt{\left(\frac{\mathrm{d}f_1(\eta)}{\mathrm{d}\eta}\right)^2+\left(\frac{\mathrm{d}f_2(\eta)}{\mathrm{d}\eta}\right)^2}_{|\eta=1} = \sqrt{0^2+\left(\frac{\mathrm{d}f_2(\eta)}{\mathrm{d}\eta}\right)^2}_{|\eta=1}$$

$$= \left|\frac{\mathrm{d}f_2(\eta)}{\mathrm{d}\eta}\right|_{|\eta=1}.$$

From this result and equation (9), it follows that

$$\left|\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{kp}\right)\right|_{|\eta=1} = \left|\frac{\mathrm{d}}{\mathrm{d}\eta}\left|\left(\frac{v}{kp}\right)\right|\right|_{|\eta=1}.$$

Thus, from the latter result and equation (5), it follows that the absolute value of the slope of the radial velocity profiles that are shown in Figures 2 and 4 of reference [1] must be equal to

$$\left|\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{kp}\right)\right| = \left|\frac{\mathrm{d}}{\mathrm{d}\eta}\left|\left(\frac{v}{kp}\right)\right|\right| = |-\mathrm{i}| = 1 \quad \text{at } \eta = 1. \tag{10}$$

As can be seen in the Figures 2 and 4 of reference [1], the deviation of the absolute value of the slope of the radial velocity profiles (including the exact solution [2]) from unity is about 30% at $\eta = 1$.

REFERENCES

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AUTHOR'S REPLY

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I have checked all formulae and programs that were used in my article [1] as well as the paper by Denisov and Khitrik [2]. The result is as follows

(1) Reference [2] has no fault except for the reference citation of Zwikker and Kosten [3]. I suppose that they referred to Tijdeman's paper [4] because there appears no explicit equation on this matter. If Denisov and Khitrik had referred to equation (B.20) in reference [4], then they have used the wrong one. The right-hand side of equation (B.20) should be multiplied by $1/\gamma$ and, in the third term of the right-hand side, η in the denominator should be replaced by n. However, equation (B.14) is correct. Their argument about the difference in slope by a factor γ might stem from this fact.

(2) It is found that the equations in my paper [1] have no fault.

(3) Figures 2(b) and (4) were calculated correctly. The following tables show the calculation results near the boundary. In all calculations, the number of points for